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A RIEMANN-PROBLEM-BASED APPROACH FOR STEADY INCOMPRESSIBLE FLOWS

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ABSTRACT

A new approach for solving steady incompressible Navier-Stokes equations is presented in this paper. This method extends the upwind Riemann-problem-based techniques to viscous flows, which is obtained by applying modified artificial compressibility Navier-Stokes equations and fully discrete high-order numerical schemes for systems of advection-diffusion equations. In this approach, utilizing the local Riemann solutions the steady incompressible viscous flows can be solved in a similar way to that of inviscid hyperbolic conservation laws. Numerical experiments on the driven cavity problem indicate that this approach can give satisfactory solutions.

KEY WORDS Upwind high-order Riemann-problem Fully discrete Incompressible viscous flow

INTRODUCTION

Incompressible flows cover a wide variety of applications which include fluid motion of water and low speed air. Therefore the solution of the incompressible Navier-Stokes equations is of great interest in industry. The major difficulty for computation of incompressible flows lies in the absence of the time derivative of density. Thus, the pressure term cannot be explicitly updated with the velocity. Hence time-dependent methods suitable for compressible equations cannot be applied without adaptation.

In order to overcome the problem, one approach is to use the artificial compressibility method which was initially introduced by Chorin[1] as a novel approach to solve steady-state viscous incompressible flow problems. The principle of this method is to replace the divergence-free continuity equation by a pseudo-time-dependent equation with a pseudo-time derivative of the pressure which is designed to vanish as steady state is reached, where the divergence-free condition is satisfied. One remarkable feature of the approach is that the pseudo-compressible Navier-Stokes equations possess a sub-problem of hyperbolic character with pseudo-pressure waves propogating with finite speed. It is this hyperbolic nature that enables us to take advantage of the robust, upwind, Riemann-problem-based techniques.

In this approach, the pseudo-compressible Navier-Stokes equations can be decoupled in terms of the advection part and the diffusion part. Then the advection part can be solved by applying a upwind, Riemann-problem-based hyperbolic scheme and then the diffusion part can be solved by another stage using a viscous scheme. This technique is called advection-diffusion operator splitting.

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In this paper, an alternative approach is proposed. Instead of using a two-stage scheme, i.e. the advection-diffusion operator splitting, the new approach solves the steady incompressible viscous fluid flows in one stage. This approach is based on two techniques. First, the pseudo-compressible Navier-Stokes equations are modified in such a way that they can be totally decoupled into a set of advection-diffusion equations. Second, the conservative upwind high-order numerical schemes used are developed on full discretization of a model advection-diffusion equation. The feature of the approach is that the steady incompressible Navier-Stokes equations are solved in a similar manner as hyperbolic conservation laws. Therefore all techniques suitable for hyperbolic systems can be similarly extended to the incompressible viscous system. To illustrate the approach, the numerical experiments of a driven cavity problem proposed by Shih[2] are presented. The numerical results computed by schemes up to fourth-order agree with the exact solution well.

This paper is orgnized as follows: First, modified artificial compressible Navier-Stokes equations are presented. Next, linear advection-diffusion systems are discussed and fully discrete high-order numerical schemes for the linear systems are given. The discussion is then extended to non-linear advection-diffusion systems, the numerical solutions to the driven cavity problem are presented and, finally, conclusions are drawn.

ARTIFICIAL COMPRESSIBILITY NAVIER-STOKES EQUATIONS

The system of incompressible Navier-Stokes equations in a vector form reads

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \nabla p = \frac{1}{Re^+}\nabla^2 u \tag{1}$$

$$\nabla \cdot u = 0 \tag{2}$$

where u, p and t are the velocity, pressure and real time, respectively, which are non-dimensionlized by a characteristic length and velocity. Re^+ is the characteristic Reynolds number.

To apply the artificial compressibility method, the continuity equation (2) is replaced by the following pseudo-time-dependent equation[1]:

$$\frac{\partial p}{\partial t} + \delta^2 \nabla \cdot u = 0 \tag{3}$$

where δ is the constant artificial compressibility parameter. Note that in such a definition the time t in the pseudo-compressibility system (1) and (3) has become a pseudo-time. Thereby, the solution of the transient behaviour loses its physical meaning until the steady state is approached asymptotically in the pseudo-time, where the time derivatives in the pseudo-system vanish so that the divergence-free velocity is satisfied.

For a reason which will be explained in a due course, equation (3) is modified to the following form:

$$\frac{\partial p}{\partial t} + \delta^2 \nabla \cdot u = \frac{1}{Re^+}\nabla^2 p \tag{4}$$

It is known that for flows with low Reynolds number the pressure approximately satisfies the Laplace equation, i.e. $\nabla^2 p = 0$, while for relatively large Reynolds number flows the magnitude of coefficient $1/Re^+$ is small. Therefore when steady state is reached the right-hand side of the equation is always very small.

Combining equation (1) with equation (4), the two-dimensional pseudo-compressible Navier-Stokes equations in Cartesian co-ordinates take on the following form:

$$U_t + F(U)_x + G(U)_y = N\,\nabla^2 U \tag{5}$$

$$U = \begin{pmatrix} p \\ u \\ v \end{pmatrix}, F(U) = \begin{pmatrix} \delta^2 u \\ u^2 + p \\ uv \end{pmatrix}, G(U) = \begin{pmatrix} \delta^2 v \\ uv \\ v^2 + p \end{pmatrix} \tag{6}$$

$$N = \begin{pmatrix} \frac{1}{Re^+} & 0 & 0 \\ 0 & \frac{1}{Re^+} & 0 \\ 0 & 0 & \frac{1}{Re^+} \end{pmatrix} \tag{7}$$

Here u and v are the velocity components in x and y direction respectively.

One way of treating multi-dimensional problems is to apply the method of fractional steps or operator splitting[3]. In this approach, the viscous system (5) can be split into two augmented one-dimensional viscous systems:

$$U_t + F(U)_x = N\, U_{xx} \tag{8}$$

and

$$U_t + G(U)_y = N\, U_{yy} \tag{9}$$

then the solution of (5) can be obtained by solving equations (8) and (9) using "Strang splitting"[3]. Therefore it is reasonable that from now on our attention is focused on studying the system of one-dimensional advection-diffusion equations.

LINEAR ADVECTION-DIFFUSION SYSTEMS

In the last section the non-linear parabolic system was derived. In order to understand the solution structure of the system and justify the Riemann-problem-based approach, for convenience the initial-value problem for a one-dimensional linear advection-diffusion system with constant coefficients is first studied.

$$\begin{gathered} U_t + AU_x = N\, U_{xx} \\ U(x,0) = U_0(x) \end{gathered} \tag{10}$$

where, U are vector functions of m conserved variables, $A = F'(U)$ is an m by m constant matrix, and N is an m by m diagonal diffusive coefficient matrix.

If we ignore the viscous terms on the right-hand side of equation (10), then equation (10) reduces to a system of conservation laws with only advective flux function $F(U) = AU$. We know that the system is hyperbolic if A is diagonalizable with real eigenvalues.

Characteristic Variables

The linear systems can be decoupled into m independent scalar equations in terms of the characteristics variables which is defined by:

$$V = R^{-1}U \tag{11}$$

However, with the viscous terms on the right-hand side of equation (10) the system is parabolic. Nevertheless, the definition of equation (11) is still adopted and the viscous system (10) is transformed into characteristic variables by multiplying equation (10) by R^{-1}:

$$R^{-1}U_t + \Lambda R^{-1}U_x = R^{-1}N\, U_{xx} \tag{12}$$

i.e.

$$V_t + \Lambda V_x = N\, V_{xx} \tag{13}$$

since $R^{-1} N R = N$. Note that the viscous system (10) is absolutely decoupled into m independent scalar advection-diffusion equations:

$$\begin{cases} v_t^{(p)} + \lambda^{(p)} v_x^{(p)} = \nu v_{xx}^{(p)} \\ p = 1, 2, \ldots, m \end{cases} \tag{14}$$

Recall that in the artificial compressibility equation (5) $\nu = 1/Re^+$, this is why the artificial compressibility equation is modified in the form of equation (4).

Applying the fully discrete numerical schemes introduced in Shi[4] the solutions of $v^{(p)}(x, t)$ for each of these scalar advection-diffusion equations can be computed. Then the original solution of equation (10) can be transformed back via:

$$U(x, t) = RV(x, t) \tag{15}$$

i.e.

$$U(x, t) = \sum_{p=1}^{m} v^{(p)}(x, t)\, r^{(p)} \tag{16}$$

The analysis procedure and the solution (16) look much like those of hyperbolic conservation laws except that the values of $v^{(p)}$ no longer hold constant along characteristic lines for each scalar equation, in other words they no longer simply advect with the initial values since:

$$\begin{aligned} \frac{dv^{(p)}}{dt} &= v_t^{(p)} + \frac{dx}{dt} v_x^{(p)} \\ &= v_t^{(p)} + \lambda^{(p)} v_x^{(p)} \\ &= \nu v_{xx}^{(p)} \end{aligned} \tag{17}$$

Instead of the right-hand side being zero, it is $\nu v_{xx}^{(p)}$. This means that, apart from advection, the values of $v^{(p)}$ are diffused with time evolution at the rate of $\nu v_{xx}^{(p)}$. However, the characteristic curve is still a straight line since:

$$\frac{dx}{dt} = \lambda^{(p)} \tag{18}$$

i.e.

$$x = \lambda^{(p)} t + x_0$$

where $x = x_0$ when $t = 0$.

Therefore the solution of equation (16) can be viewed as the superposition of m waves, each of which is not only advected independently with propagating speed $\lambda^{(p)}$, but also diffused independently at the dissipating rate $\nu v_{xx}^{(p)}$.

Fully discrete high-order schemes for linear systems

In this section the fully discrete conservative high-order schemes for linear advection-diffusion systems are presented. The detailed derivation of the schemes can be found in Shi[4]. Conservative numerical schemes are considered which have the following form:

$$U_j^{n+1} = U_j^n - \frac{k}{h}(F_{j+1/2} - F_{j-1/2}) \tag{19}$$

where h is a cell width and k a time step. The numerical fluxes $F_{j+1/2}$ are defined as follows.

Three-point centered numerical flux for systems. The three-point fully discrete numerical flux for systems, which has first-order accuracy in time and second-order in space, i.e. order (1, 2), can be written as:

$$F_{j+\frac{1}{2}} = \frac{1}{2}(F_j^n + F_{j+1}^n) - \frac{1}{2}\sum_{p=1}^{m} |\lambda_{j+\frac{1}{2}}^{(p)}|\, \alpha_{j+\frac{1}{2}}^{(p)}\, r_{j+\frac{1}{2}}^{(p)}$$

$$+ \sum_{p=1}^{m}\left(\frac{1}{2} - 1/|Re_{j+\frac{1}{2}}^{(p)}| - |c_{j+\frac{1}{2}}^{(p)}|/2\right)|\lambda_{j+\frac{1}{2}}^{(p)}|\, \alpha_{j+\frac{1}{2}}^{(p)}\, r_{j+\frac{1}{2}}^{(p)} \qquad (20)$$

where $c_{j+1/2}^{(p)} = \lambda_{j+1/2}^{(p)} k/h$ is a cell Courant number; $Re_{j+1/2}^{(p)} = \lambda_{j+1/2}^{(p)} h/b$ is a cell Reynolds number and $\alpha_{j+1/2}^{(p)} = \Delta v_{j+1/2}^{(p)}$ is called the wave strength across the pth wave.

Notice that the above flux (20) not only includes the advection flux, $(1/2 - |c_{j+1/2}^{(p)}|/2)\, \Delta F_{j+1/2}^{(p)}$, but also involves the diffusion flux, $(-1/|Re_{j+1/2}^{(p)}|)\, \Delta F_{j+1/2}^{(p)}$. Flux (20), therefore, is an advection-diffusion numerical flux.

Five-point upwind-based numerical flux for systems. The advection-diffusion numerical flux of five-point scheme (order (1, 3)) for systems can be written as:

$$F_{j+\frac{1}{2}} = \frac{1}{2}(F_j^n + F_{j+1}^n) - \frac{1}{2}\sum_{p=1}^{m} |\lambda_{j+\frac{1}{2}}^{(p)}|\, \alpha_{j+\frac{1}{2}}^{(p)}\, r_{j+\frac{1}{2}}^{(p)}$$

$$+ \sum_{p=1}^{m}\left(D_{j+\frac{1}{2}}^{(p)}|\lambda_{j+\frac{1}{2}}^{(p)}|\, \alpha_{j+\frac{1}{2}}^{(p)}\, r_{j+\frac{1}{2}}^{(p)} + D_{j+L+\frac{1}{2}}^{(p)}|\lambda_{j+L+\frac{1}{2}}^{(p)}|\, \alpha_{j+L+\frac{1}{2}}^{(p)}\, r_{j+L+\frac{1}{2}}^{(p)}\right) \qquad (21)$$

where

$$D_{j+\frac{1}{2}}^{(p)} = \frac{1}{6}\left(c_{j+\frac{1}{2}}^{(p)}\right)^2 + \left(1/|Re_{j+\frac{1}{2}}^{(p)}| - \frac{1}{2}\right)|c_{j+\frac{1}{2}}^{(p)}| + \frac{1}{3} - 1/|Re_{j+\frac{1}{2}}^{(p)}|$$

$$D_{j+L+\frac{1}{2}}^{(p)} = \frac{1}{6} - c_{j+L+\frac{1}{2}}^{(p)}/Re_{j+L+\frac{1}{2}}^{(p)} - \left(c_{j+L+\frac{1}{2}}^{(p)}\right)^2$$

$$\begin{cases} L = -1 \text{ if } c_{j+\frac{1}{2}}^{(p)} > 0 \\ L = 1 \text{ if } c_{j+\frac{1}{2}}^{(p)} < 0. \end{cases}$$

Five-point centred numerical flux for systems. The five-point advection-diffusion numerical flux (order (2, 4)) for systems can be written as:

$$F_{j+\frac{1}{2}} = \frac{1}{2}(F_j^n + F_{j+1}^n) - \frac{1}{2}\sum_{p=1}^{m} |\lambda_{j+\frac{1}{2}}^{(p)}|\, \alpha_{j+\frac{1}{2}}^{(p)}\, r_{j+\frac{1}{2}}^{(p)}$$

$$+ \sum_{p=1}^{m}\left(D_{j+\frac{1}{2}}^{(p)}\, \lambda_{j+\frac{1}{2}}^{(p)}\, \alpha_{j+\frac{1}{2}}^{(p)}\, r_{j+\frac{1}{2}}^{(p)}\right)$$

$$+ D_{j+L+\frac{1}{2}}^{(p)}\, \lambda_{j+L+\frac{1}{2}}^{(p)}\, \alpha_{j+L+\frac{1}{2}}^{(p)}\, r_{j+L+\frac{1}{2}}^{(p)}$$

$$+ D_{j+M+\frac{1}{2}}^{(p)}\, \lambda_{j+M+\frac{1}{2}}^{(p)}\, \alpha_{j+M+\frac{1}{2}}^{(p)}\, r_{j+M+\frac{1}{2}}^{(p)} \qquad (22)$$

where

$$D_{j+L+\frac{1}{2}}^{(p)} = \frac{1}{12} + \frac{1}{12|Re_{j+L+\frac{1}{2}}^{(p)}|} + \left(\frac{1}{24} - \frac{1}{2|Re_{j+L+\frac{1}{2}}^{(p)}|} - \frac{1}{2(Re_{j+L+\frac{1}{2}}^{(p)})^2}\right)$$

$$|c_{j+L+\frac{1}{2}}^{(p)}| - \left(\frac{1}{12} + \frac{1}{2|Re_{j+L+\frac{1}{2}}^{(p)}|}\right)(c_{j+L+\frac{1}{2}}^{(p)})^2 - \frac{|(c_{j+L+\frac{1}{2}}^{(p)})^3|}{24}$$

$$D^{(p)}_{j+\frac{1}{2}} = \frac{1}{2} - \frac{7}{6\,|Re^{(p)}_{j+\frac{1}{2}}|} + \left(\frac{1}{(Re^{(p)}_{j+\frac{1}{2}})^2} - \frac{7}{12}\right)|c^{(p)}_{j+\frac{1}{2}}| + \frac{(c^{(p)}_{j+\frac{1}{2}})^2}{|Re^{(p)}_{j+\frac{1}{2}}|} + \frac{|(c^{(p)}_{j+\frac{1}{2}})^3|}{12}$$

$$D^{(p)}_{j+M+\frac{1}{2}} = \frac{1}{12\,|Re^{(p)}_{j+M+\frac{1}{2}}|} - \frac{1}{12} + \left(\frac{1}{24} + \frac{1}{2\,|Re^{(p)}_{j+M+\frac{1}{2}}|} - \frac{1}{2(Re^{(p)}_{j+M+\frac{1}{2}})^2}\right)$$
$$|c^{(p)}_{j+M+\frac{1}{2}}| + \left(\frac{1}{12} - \frac{1}{2\,|Re^{(p)}_{j+M+\frac{1}{2}}|}\right)(c^{(p)}_{j+M+\frac{1}{2}})^2 - \frac{|(c^{(p)}_{j+M+\frac{1}{2}})^3|}{24}$$

$$\begin{cases} L = -1, M = 1 & \text{if } c^{(p)}_{j+1/2} > 0 \\ L = 1, \quad M = -1 & \text{if } c^{(p)}_{j+1/2} < 0. \end{cases}$$

NON-LINEAR ADVECTION-DIFFUSION SYSTEMS

In this section the discussion is extended to a non-linear system of advection-diffusion equations

$$U_t + F(U)_x = N\,U_{xx} \tag{23}$$

where, $U(x, t)$ is the column vector of m conserved variables; $F(U)$ is a vector-valued physical advection flux function of m components; N is a diagonal diffusive coefficient matrix.

It is said that the parabolic system of equation (23) has hyperbolic character if the m by m Jocabin matrix

$$A(U) = F'(U) \tag{24}$$

is diagonalizable with real eigenvalues $\lambda^{(1)}(U), \lambda^{(2)}(U), \ldots, \lambda^{(m)}(U)$.

Let us take the one-dimensional artificial compressibility Navier-Stokes equations (8) introduced in the second section for example. The eigenvalues of the Jocabin matrix $F'(U)$ of equation (8) are:

$$\lambda^{(1)}(U) = u - a, \lambda^{(2)}(U) = u, \lambda^{(3)}(U) = u + a \tag{25}$$

where

$$a = \sqrt{u^2 + \delta^2} \tag{26}$$

is analogous to the sound speed in the Euler equations. The corresponding right eigenvectors are:

$$r^{(1)}(U) = \begin{pmatrix} -(a+u) \\ 1 \\ -v/a \end{pmatrix}, r^{(2)}(U) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, r^{(3)}(U) = \begin{pmatrix} a-u \\ 1 \\ v/a \end{pmatrix} \tag{27}$$

The eigenvalues are not constant. The strategy for solving systems of the non-linear equations is to utilize the Riemann problem solutions. Therefore Riemann solvers for the artificial compressibility Navier-Stokes equations are needed. Following Roe's[5] strategy, the eigenvalues and eigenvectors are evaluated at the average state $\bar{U}$. For the two-dimensional case it takes the following form

$$\bar{u} = \tfrac{1}{2}(u_j + u_{j+1}), \bar{v} = \tfrac{1}{2}(v_j + v_{j+1}), \bar{a} = (\bar{u}^2 + \delta^2)^{1/2} \tag{28}$$

The wave strengths $\bar{\alpha}^{(p)}$ are determined by:

$$\begin{cases} \bar{\alpha}^{(1)} = \Delta p(\bar{u} + \bar{a})/2\bar{a}\delta^2 - \Delta u/2\bar{a} \\ \bar{\alpha}^{(2)} = \Delta v - \Delta p\bar{v}/\bar{a}^2 - \Delta u\bar{v}\bar{u}/\bar{a}^2 \\ \bar{\alpha}^{(3)} = \Delta u/2\bar{a} - \Delta p(\bar{u} - \bar{a})/2\bar{a}\delta^2 \end{cases} \tag{29}$$

where

$$\Delta u = u_{j+1} - u_j,\ \Delta v = v_{j+1} - v_j,\ \Delta p = p_{j+1} - p_j$$

For splitting system (9) in the y direction, the corresponding eigenvalues and eigenvectors can be obtained by interchanging the roles of u and v.

From the observation of the eigenvalues (25) we know that the eigenvalue $\lambda^{(1)}$ is always negative and the $\lambda^{(3)}$ always positive, i.e. there is no sonic point in the Riemann solution. This is indeed a very pleasant feature of the artificial compressibility Navier-Stokes equations because there is no fear of failure caused by entropy-violating when applying an approximate Riemann solver.

APPLICATION TO THE DRIVEN CAVITY PROBLEM

Because incompressible viscous flow in a driven cavity has a simple geometry and can present some interesting complex fluid dynamic features, such as vortex development and boundary-layer formation (high Reynolds number) near the walls of the cavity, the driven cavity problem has been accepted as a ideal test problem for evaluating an optimum numerical scheme for solving the incompressible Navier-Stokes equations[6]. The specific driven cavity problem considered in this paper is proposed by Shih and Tan[2] Instead of the classical lid-driven cavity flow, this flow is driven by combined shear and body forces. The advantage of Shih's problem is that the exact solution is known, therefore provides a reliable base with which to compare.

To illustrate the approach, Reynolds number $Re^+ = 100$ and $Re^+ = 500$ were chosen for the tests. The 1×1 domain is discretized with 40×40 uniform cells. The Riemann solver (28)-(29) is applied to carry out these tests. The pre-numerical experiments indicate that the artificial compressibility parameter δ affects not only the convergence rate but also the accuracy of the solution[7]. When determining the value of δ, the first priority is put on the accuracy and then the convergence rate.

Figure 1 shows the exact solutions which are presented for comparison with the numerical results. In *Figure 1*, (a) presents contours of isovelocity u; (b) presents the isovelocity v; (c) is contours of isopressure for $Re^+ = 100$, and (d) is contours of isopressure for $Re^+ = 500$.

We are interested in a steady-state solution. After finite iterations the artificial compressibility approach will approximately converge to the steady-state solution. The residual error is measured by a parameter, which monitors the convergence rate of the solution to the steady-state. One parameter introduced by Peyret and Taylor[8] have the following form:

$$R = \frac{1}{N}\sum_{i,j}\left|U_{i,j}^{n+1} - U_{i,j}^{n}\right| \tag{30}$$

where N is the number of computational cells; U is the flow variables.

If the magnitude of R is less than a preset value, the solution is said to have converged to the stead-state solution. In the driven cavity simulation the parameter (30) was used. The convergence criterion or the average residual value is set to be 1×10^{-5}.

Case 1: $Re^+ = 100$

Figure 2 shows the numerical solutions by the three-point centred scheme (20). The artificial compressibility parameter $\delta = 2$ was used in the case. The convergence rate is fast: the CPU time used is 7.58 minutes (DECstation 5,000/200). In *Figure 2*, (a) is the isovelocity u; (b) is the isovelocity v; (c) presents the computed isopressure, and (d) shows the comparison between the

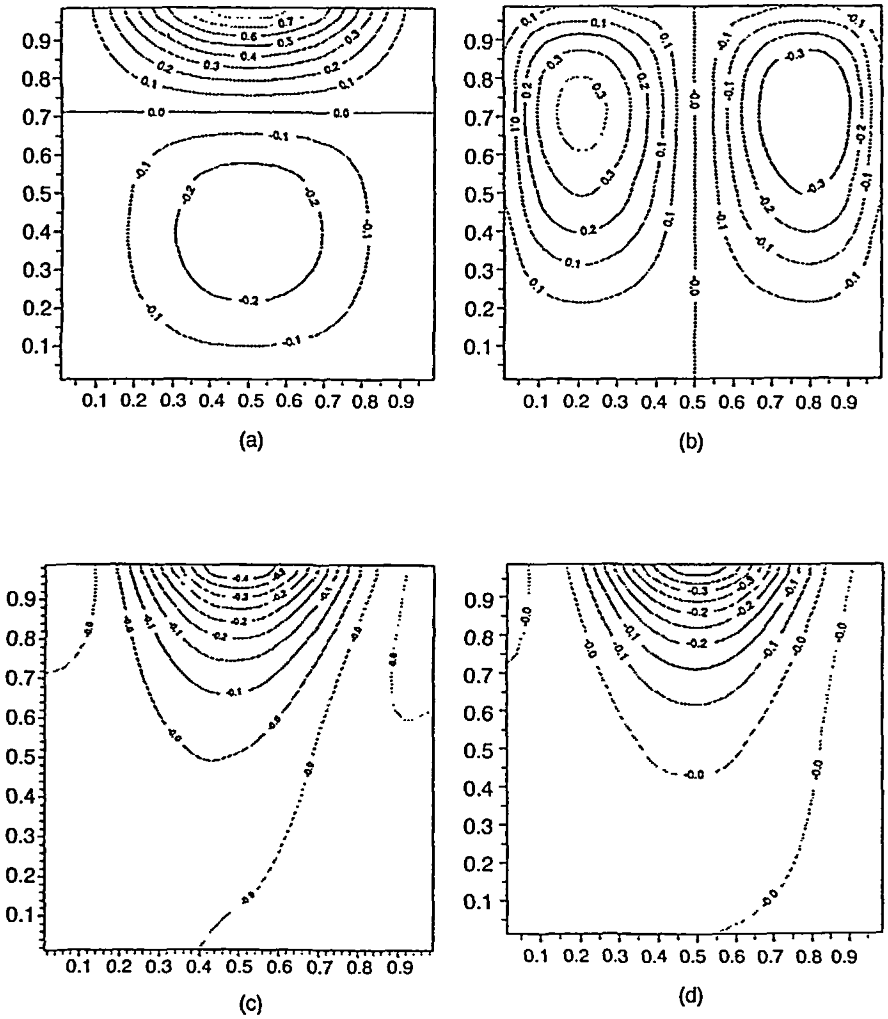

Figure 1 The exact solution of the driven cavity problem: (a) Velocity u; (b) Velocity v; (c) Pressure for $Re^+ = 100$; (d) Pressure for $Re^+ = 500$

exact solution (line) and the numerical solution (symbol) for the velocity v at $y = 0.5$. As shown in the figures the numerical results have good agreement with the exact solutions.

Figure 3 shows the numerical solutions by the five-point centred scheme (22). The arrangement of the figure is the same as *Figure 2*. Comparing with the three-point scheme, the five-point scheme used nearly twice the CPU time as that of the the three-point scheme, which is because of the complexity of the algebra of the scheme. Again there is a good agreement between the numerical results and the exact solutions.

Case 2: $Re^+ = 500$

As in case 1, *Figures 4* and *5* show the numerical solutions by the three and five-point centred schemes, respectively. In this case, the artificial compressibility parameter $\delta = 1$ was used. Again the numerical results of velocity and pressure have good agreements with the exact solution.

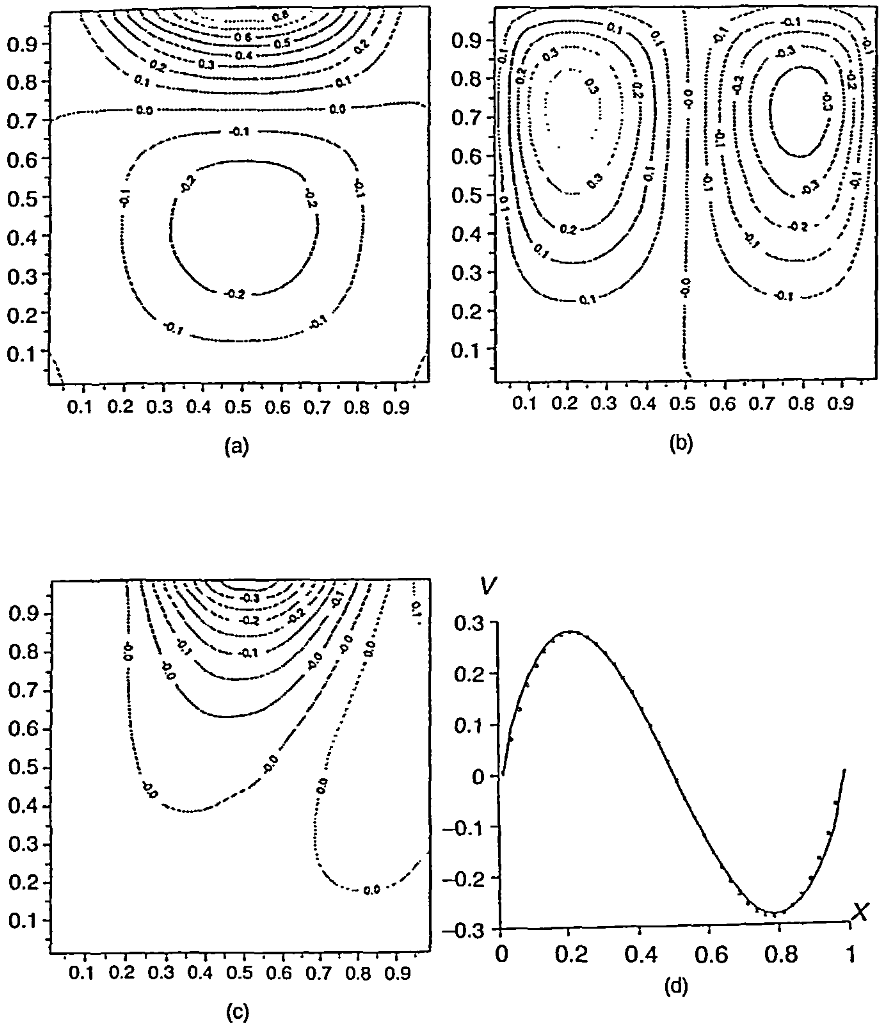

Figure 2 Numerical solutions by the three-point scheme for $Re^{+} = 100$: (a) Velocity u; (b) Velocity v; (c) Pressure; (d) Comparison between the exact solution (line) and the numerical solution (symbol) for the velocity v at $y = 0.5$

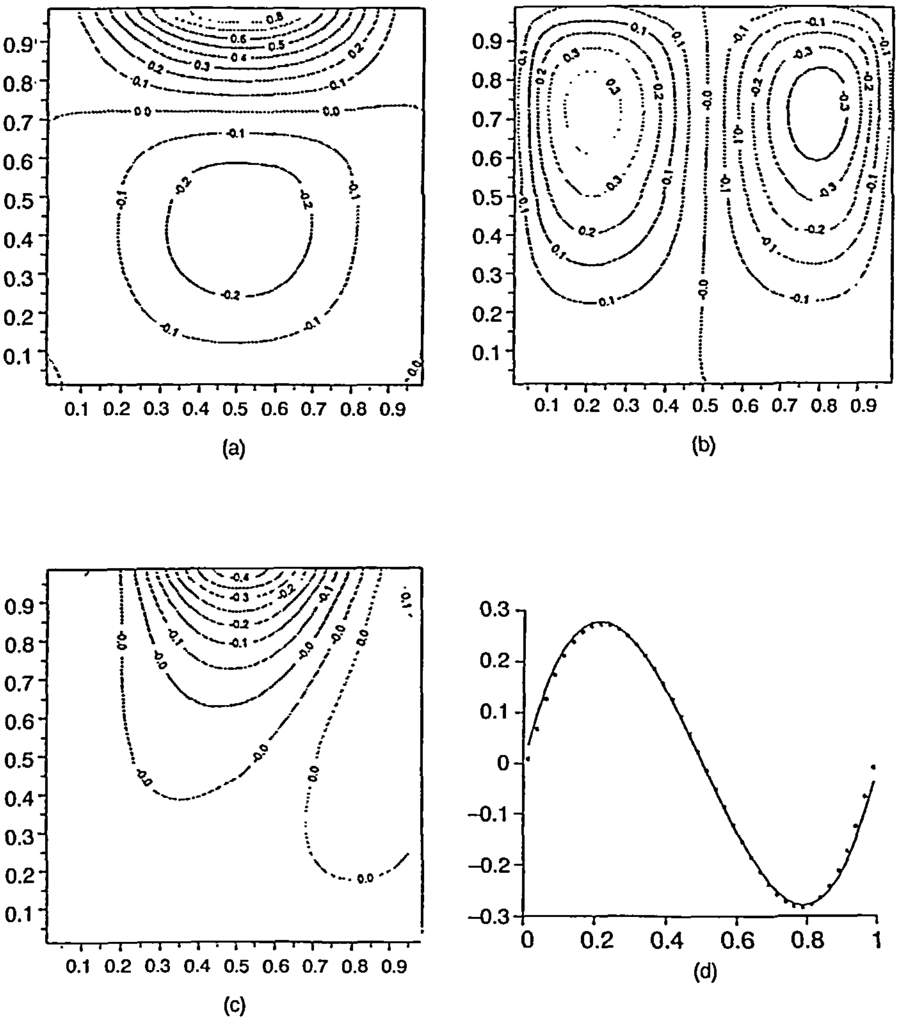

Figure 3 Numerical solutions by the five-point scheme for $Re^+ = 100$: (a) Velocity u; (b) Velocity v; (c) Pressure; (d) Comparison between the exact solution (line) and the numerical solution (symbol) for the velocity v at $y = 0.5$

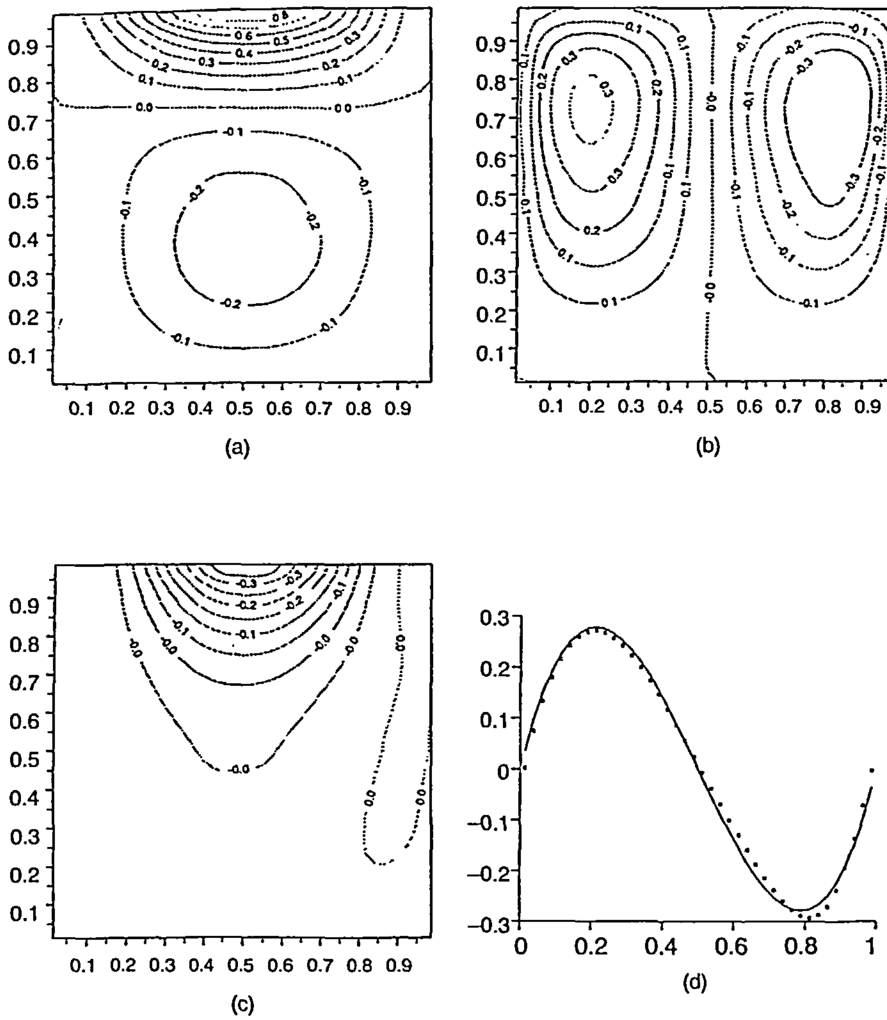

Figure 4 Numerical solutions by the three-point scheme for $Re^{+} = 500$: (a) Velocity u; (b) Velocity v; (c) Pressure; (d) Comparison between the exact solution (line) and the numerical solution (symbol) for the velocity v at $y = 0.5$

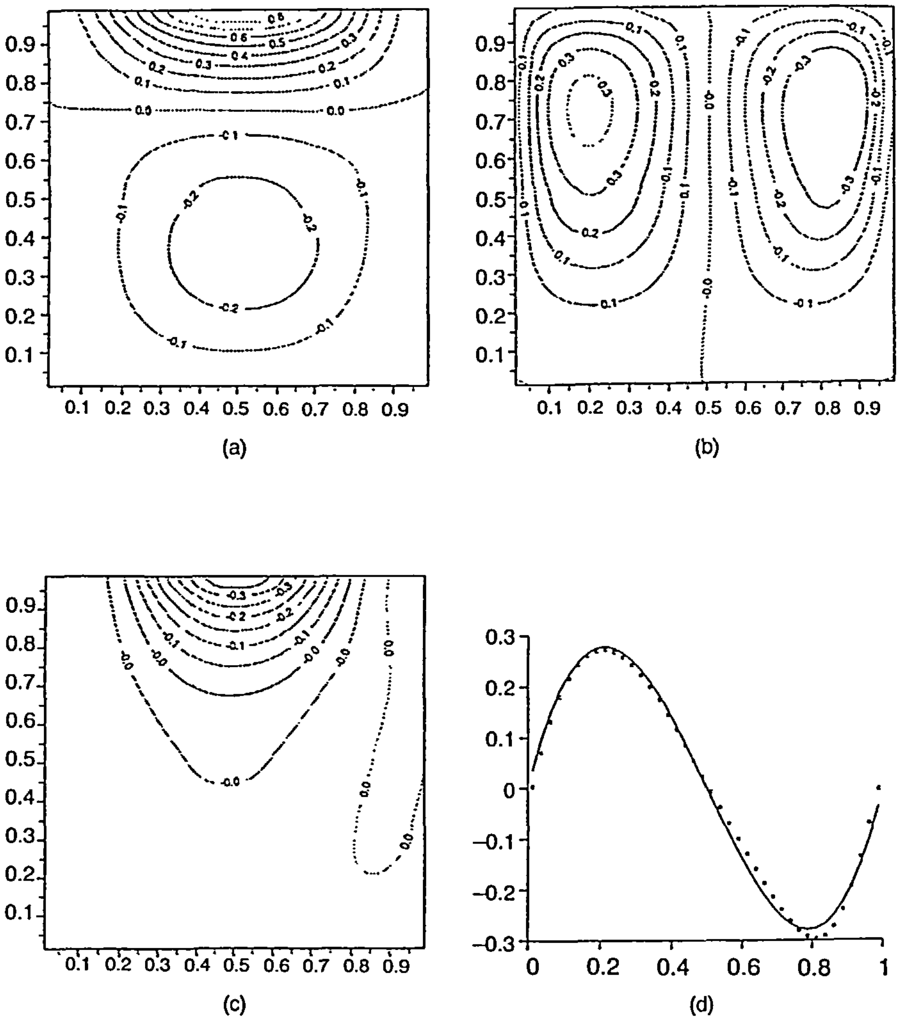

Figure 5 Numerical solutions by the five-point scheme for $Re^{+} = 500$: (a) Velocity u; (b) Velocity v; (c) Pressure; (d) Comparison between the exact solution (line) and the numerical solution (symbol) for the velocity v at $y = 0.5$

CONCLUSIONS

Distinguishing from the traditional numerical methods, this paper extends the upwind Riemann-problem-based technique to the solution of steady incompressible Navier-Stokes equations. This approach is based on two techniques: the modified pseudo-compressible Navier-Stokes equations which can be totally decoupled into a set of advection-diffusion equations and the development of fully discrete high-order conservative numerical schemes for systems of advection-diffusion equations. This approach has been tested and validated by solving two-dimensional driven cavity problems with Reynolds number Re^+ up to 500.

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ERRATUM

Volume 6 Number 3, the May issue of this journal, included the following papers: The Application of Split-Coefficient Matrix Method to Transient Two Phase Flows by D.M. Lu, H.C. Simpson and A. Gilchrist; and Conjugate Heat Transfer Analysis of Fluid Flow in a Phase-Change Energy Storage Unit by Hongjun Li, C.K. Hsieh and D.Y. Goswami.

Within the body of the issue the authors' names were correctly attributed to each article but on the Contents Page both articles were attributed to the first group of authors. The Publisher apologizes for this error.